# Financial Deleveraging and the International Transmission of Shocks*

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## Abstract

The recent financial crisis has highlighted the role of interdependence among major economies through linkages among financial institutions, in addition to the trade linkages that are at the centre of traditional models of the international business cycle. This paper develops a model of the international transmission of shocks through de-leveraging across financial institutions. In a macro-economic model in which highly levered investors hold interconnected portfolios across countries, we show that the presence of binding leverage constraints introduces a powerful financial transmission channel which results in a high correlation among macroeconomic aggregates during business cycle downturns, quite independent of the size of international trade linkages.

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## 1. Introduction

The current global financial crisis has highlighted the critical role of financial markets in the international propagation of business cycle shocks, both in transmitting shocks from one country to another and in magnifying the effects of shocks. A relatively minor (on a global scale) deterioration in the US subprime mortgage market led to a much larger collapse in the asset values of major US investment banks, which in turn quickly precipitated a major world crisis affecting financial institutions across the globe.

It is widely agreed that high financial leverage – high ratios of assets to underlying capital – was a critical factor in the magnifying effects of the crisis. As asset values declined, highly levered financial institutions found their net worth sharply eroded, and were forced to shed assets to avoid unacceptable risks of insolvency. This process of 'de-leveraging' drove asset values down further, in turn adversely impairing the balance sheets of other institutions. While the financial dynamics of balance sheet deleveraging have been widely discussed elsewhere, it is less well understood how this process affects macroeconomic outcomes, and that financial deleveraging alone may generate an immediate and powerful international transmission of shocks. A clear pre-requisite for such a response to deleveraging would seem to be the presence of some type of financial frictions or distortions in credit markets. After all, in a Modigliani-Miller world, leverage is irrelevant. Thus, in order to capture the dynamics of the financial meltdown, it financial frictions will be of critical importance.

In the context of the international transmission of business cycles, however, other puzzles arise. Most models of business cycle transmission still rely on the international linkage of countries through trade flows. While global trade has been growing at remarkable rates over the past two decades, it is still the case that the major world regions – the United States, Asia, and

Europe, are to a large extent 'closed' economies, with the export share from one region to another representing only a small proportion of overall GDP. This perspective led many to believe in the prospects for a 'de-coupling' of the rest of the world from a US recession. But the rapid deterioration in economic activity in almost all regions of the world during the current crisis appears to be much larger than would have been anticipated based on trade linkages alone. Krugman (2008) suggests that traditional multi-country business cycle models lack a critical 'international finance multiplier', by which financial shocks in one country affect investment both in the original country and in other countries that have financial or balance sheet linkages with the original country.

This paper develops a theoretical model of a balance sheet channel for the international transmission of shocks. The model emphasizes how a process of financial de-leveraging generated by a downturn in one country is spread around the globe through interconnected portfolios. In the presence of leverage constraints, we show that this gives rise to a separate financial transmission mechanism of business cycle shocks that is completely independent of trade linkages. In fact, we work with a highly stripped down 'one world good' model in which, in steady state, there are no trade linkages across countries at all.

The paper's main contribution is to compare how macro shocks are transmitted under two financial market structures. We develop a two country model in which investors borrow from savers in each country, and invest in fixed assets. Investors also diversify their portfolios across countries, and hold equity positions in the assets of the other country, as well as their own. Investors cannot commit to repay savers, however, and in order to enforce payment, may face limits on the maximum amount of leverage on their balance sheets. We look at one environment where leverage limits do not bind. In this case, the Modigliani-Miller theorem applies. We also show in this case that the international transmission of shocks is quite limited. Specifically, there is no international transmission due to ‘de-leveraging’. A negative productivity shock which leads to a fall in the value of assets in one country will cause financial institutions to sell some assets and reduce their debt exposure. But that does not affect other countries. In fact, in other countries, investors increase their borrowing. More broadly, business cycle fluctuations across countries are essentially uncorrelated in the absence of limits on leverage.

When leverage constraints are binding, however, there is a powerful transmission of shocks across countries. A fall in asset values in one country forces an immediate and large process of de-leveraging in that country’s financial institutions. But the deterioration in asset values leads to a worsening of leverage constraints in other countries as well, causing a sell-off in assets and a forced reduction in borrowing. The result is a large interactive magnification of the initial shock, a big fall in investment, and a very high correlation in business cycles across countries during the resulting downturn.

The model draws heavily on a number of separate literatures. First, and most importantly, we follow Kiyotaki and Moore (1997) in imposing leverage limits on investors. This leads to a wedge between the effective returns faced by investors and savers, and can act as an amplification mechanism for business cycle shocks. Second, we emphasize the linkages among countries through the presence of interconnected portfolios. Portfolio interlinkages, in a somewhat different context, have for some time been seen as important in the contagion effects of financial shocks (see Rigobon 2003, Pavlova and Rigobon, 2008). Finally, we introduce endogenous portfolio interdependence through the recently developed techniques of Devereux and Sutherland (2009).

The paper is organized as follows. The next section provides some evidence of the importance of financial deleveraging in the recent business cycle downturn. We then develop the basic two country model in which investors and savers interact, but investors may be limited by leverage constraints. In section 4, we explore the effects of a negative productivity shock in one country and demonstrate the role of deleveraging in the propagation of business cycle shocks across countries.

## 2. Empirical evidence

We now present some empirical evidence that supports our contention that global deleveraging may have been an important propagation mechanism for the crisis, focusing on its effect on Asia. For the major European economies and America, there is a clear transmission path from real and expected losses on sub-prime mortgages and associated derivative products, driving increasing fragility of bank balance sheets, leading to disintermediation by the financial system, and finally deleveraging of the economy more broadly. For many Asian economies, the story is radically different. Exposure of domestic banks to toxic assets is limited, and banks generally remain well capitalized. Yet, as shown in Figure 1, the region is experiencing a significant slowdown in lock-step with Europe and the United States. Our evidence below is at least suggestive that global deleveraging played a role in the propagation of the crisis to Asia.

First there is clear evidence that deleveraging by banks has reduced the supply of credit in Asia. Table 1 contains the growth rate of total short-term exposures of US banks to major Asian economies. This is the total stock among US reporting banks of all loans to the destination economy with less than one year remaining until maturity. Under normal circumstances, in any given quarter, new claims may be issued and many of the existing claims that mature may be rolled over. A rapid decline in less than one year (for example, to Chinese Taipei between

2008Q2 and 2008Q4) may be consistent with virtually no new issuance, and few existing loans being rolled over. Indeed, the average decline between 2008Q3 and 2008Q4 represents a 26% in total claims on Asia, demonstrating that US banks have substantially deleveraged their balance sheets with respect to Asia since the beginning of the crisis.

Table 1. Short term claims of US banks on Asian economies, $US millions.

| Destination of Funds | 2007Q4 | 2008Q1 | 2008Q2 | 2008Q3 | 2008Q4 |
|---|---|---|---|---|---|
| Hong Kong | 10,079 | 10,066 | 12,900 | 11,366 | 8,837 |
| Singapore | 17,007 | 16,966 | 15,196 | 11,778 | 10,188 |
| China | 13,192 | 11,635 | 14,795 | 12,693 | 6,498 |
| Chinese Taipei | 7,845 | 9,689 | 8,929 | 7,155 | 3,795 |
| India | 25,722 | 20,779 | 16,582 | 17,093 | 13,801 |
| Indonesia | 6,007 | 5,902 | 5,286 | 6,782 | 5,313 |
| Malaysia | 3,345 | 3,431 | 4,054 | 2,201 | 1,997 |
| Philippines | 1,370 | 2,060 | 1,923 | 1,579 | 1,547 |
| South Korea | 26,254 | 27,435 | 28,027 | 29,873 | 21,518 |
| Thailand | 794 | 860 | 534 | 692 | 869 |

Source: BIS International Banking Statistics

Aside from bank balance sheets, we can also find clear evidence consistent with deleveraging in other instruments. Equities in particular were believed by many policymakers to be a vector of contagion, as the following quote by Rakesh Mohan, Deputy Governor of the Reserve Bank of India, indicates:

"Our problems are mainly due to the sell-off by foreign institutional investors in the domestic equity markets leading to a sharp reduction in net capital inflows and the sharp slowdown in global economic activity and external demand." (Mohan 2009)

This view is consistent with the data on international capital flows captured by the Treasury International Capital System (see Figure 2). The crisis has seen a fall in both inflows and outflows of capital from the US. In the model, we see this type of deleveraging, when combined with binding leverage constraints among financial institutions, can impart an independent international transmission of shocks.

## 3. The model

In this section we describe a basic two country model with levered borrowers and lenders in each country. The countries are called home and foreign. Within each country, there are investors and savers, both of whom use the same fixed asset, and have infinite horizons. Investors purchase the fixed asset and rent it to production firms, receiving a risky return in exchange. We may think of this investment as a purchase of an equity claim in the production firm. Investors are more impatient than savers, so they will borrow from savers in order to invest in the fixed asset.[1] Savers also make use of the fixed asset in home production. Savers therefore choose a portfolio in which they hold the debt of investors and the fixed asset. By assumption, savers do not hold domestic or foreign equity. Investors in either country, however, may trade claims with investors in the other country so as to diversify their portfolio of equity holdings. Thus investors in each country hold levered investments, but also have equity portfolios that are interconnected across countries. Finally, both investors and savers in each country supply labor inelastically to production firms.

### Investors

We normalize the population of each country to unity, with a measure $n$ of investors and $1-n$ savers. The representative investor in the home country maximizes:

$$E_t \sum_{s=t}^{\infty} \theta_s^I U(C_s^I) \,, \tag{1}$$

[1] Because they are more impatient than savers, investors will never accumulate enough resources to cover the cost of investment in any period.

where $C_s^I$ is consumption of the final good. To keep the analysis solely focused on financial inter-linkages between countries, it is assumed that there is just one world good. Adding an endogenous terms of trade to the analysis would enrich the response, but would not fundamentally alter the cross country transmission of de-leveraging modeled here.[2] We define the discount factor for investors such that:

$$\theta_{s+1}^I = \beta^I(\bar{C}_s^I)\theta_s^I, \quad \beta^I{}'(\bar{C}_s^I) \leq 0,$$

with $\beta^I{}'(\bar{C}_s^I) \leq 0$, where $\bar{C}_s^I$ is the economy-wide average consumption of investors. Thus the investor's time preference is increasing in consumption, but the rate of time preference is taken as given by the individual investor. The assumption of endogenous time preference for investors plays the usual role of ensuring a stationary wealth distribution among groups, both within countries and across countries. But it also plays a key role in allowing for a comparison of an economy in which financial constraints bind with one where they do not bind, as we discuss below.

Investors receive income from their current holdings of domestic and foreign equity, as well as labor income from working in the domestic production firm. In addition, they must repay their debts owed to domestic savers. They then issue new debt, purchase equity claims on home and foreign investments, and consume. The home country investor's budget constraint is written as:

(2) $$C_t^I + q_{1t}k_{1t}^I + q_{2t}k_{2t}^I = W_t^I + (q_{1t} + R_{1Kt})k_{1t-1}^I + (q_{2t} + R_{2Kt})k_{2t-1}^I + B_t^I - R_{t-1}B_{t-1}^I,$$

[2] Aghion et al. (2004) explore the importance of terms of trade movements in affecting borrowing constraints in an emerging market economy framework. Extending our model to a setting with endogenous terms of trade would affect borrowing constraints through the impact of terms of trade adjustment on net worth in a similar way to the effects of asset price changes in the present version of the model.

where $q_{1t}$ and $q_{2t}$ represent the price of the fixed asset (or equity) in the home and foreign country respectively, and $k_{1t}^I$ and $k_{2t}^I$ are the portfolio holdings of the fixed assets in each country held by the home investor. The fixed asset of the home (foreign) country earns a return of $R_{1Kt}$ ($R_{2Kt}$). $W_t^I$ is wage income for the investor, who supplies one unit of labor. Finally, $B_t^I$ is the debt issued to domestic savers, and $R_{t-1}B_{t-1}^I$ is payment on previously incurred debt.

A question may arise as to why only investors can purchase the fixed assets, which are then used by final goods firms. As in Bernanke et al. (2000), we could assume that investors (or entrepreneurs, in their model) have some special capability for transforming a unit of the fixed asset into a usable factor of production that is rented to production firms. Lenders cannot do this, and so may gain only indirectly from the investment, by lending to the investors.

In addition to constraint (2), we assume that investors face a constraint on total leverage due to an inability to commit to repayment, as in Kiyotaki and Moore (1997). Total debt is assumed to be restricted to be no greater than $\kappa$ times the market value of equity assets, where $\kappa < 1$. Thus home investors' choices are constrained by:

(3) $$B_t^I \leq \kappa(q_{1t}k_{1t}^I + q_{2t}k_{2t}^I).$$

The full leverage rate (the value of assets to capital) for investors is then $\frac{1}{1-\kappa}$ in the case where the leverage constraint (3) is binding. We take $\kappa$ as a free variable in our analysis. Leverage constraints in the form of (3) have been used quite widely in the literature on asset prices (Aiyagari and Gertler 1999), emerging market crises (Mendoza and Smith 2006), borrowing in a small open economy (Uribe 2006), and monetary policy with credit frictions (Iacoviello, 2005). Kiyotaki and Moore (1997) show that $\kappa$ may depend on the borrowing rate and expected capital gains on equity under some circumstances.

Investors in the home country choose investment in the home equity, the foreign equity, and borrowing to maximize their expected utility subject to their budget constraint (2) and leverage constraint (3), giving the conditions:

(4) $$U'(C_t^I) = E_t \beta^I (C_t^I) U'(C_{t+1}^I) \frac{(q_{1t+1} + R_{1Kt+1})}{q_{1t}} + \kappa \mu_t ,$$

(5) $$U'(C_t^I) = E_t \beta^I (C_t^I) U'(C_{t+1}^I) \frac{(q_{2t+1} + R_{2Kt+1})}{q_{2t}} + \kappa \mu_t ,$$

(6) $$U'(C_t^I) = E_t \beta^I (C_t^I) U'(C_{t+1}^I) R_t + \mu_t ,$$

where $\mu$ is the multiplier on the leverage constraint, or the utility benefit of an extra unit of debt to the investor. If this is positive, it means that the entrepreneur would like to borrow more, but is constrained by (3), and therefore current marginal utility is less than expected future marginal utility times the return on investing in either the home or foreign country. Thus $\mu$ is a measure of the value of the opportunity to make a levered investment. To show this, put (4), (5) and (6) together to obtain:

(7) $$\mu_t = E_t \beta^I (C_t^I) U'(C_{t+1}^I) \left[ \frac{\omega_t r_{1,t+1} + (1-\omega_t) r_{2,t+1} - R_t}{1-\kappa} \right],$$

where $\omega_t = \dfrac{q_{1,t} k_{1,t}}{q_{1,t} k_{1,t} + q_{2,t} k_{2,t}}$ is the portfolio share in the home equity, and $r_{1,t+1} = \dfrac{q_{1,t+1} + R_{1K,t+1}}{q_{1,t}}$ ( $r_{2,t+1} = \dfrac{q_{2,t+1} + R_{2K,t+1}}{q_{2,t}}$ ) is the return on the home (foreign) equity. Equation (7) shows that, for a given distribution of excess returns and consumption, $\mu$ is higher the higher is the leverage rate.

Note that the leverage constraint does not directly affect the investors' incentive to diversify equity holdings across countries, since (3) applies equally to borrowing for domestic or foreign equity purchases. Thus we may put (4) and (5) together to get the standard portfolio selection condition:

$$(8) \qquad E_t U'(C_{t+1}^I)\left(\frac{(q_{1t+1}+R_{1Kt+1})}{q_{1t}} - \frac{(q_{2t+1}+R_{2Kt+1})}{q_{2t}}\right) = 0 .$$

Given that the portfolio choice may be written in the form (8), we can use the recent methods described in Devereux and Sutherland (2009) to derive the optimal equity portfolio of each country's investors. This involves using a second order approximation of condition (8) in conjunction with a linear approximation of the remaining aspects of the model. We discuss the details involved in portfolio choice more fully below.

**Savers**

Savers have preferences given by:

$$(9) \qquad E_t \sum_{s=t}^{\infty} \theta_s^S U(C_s^S) .$$

Again, as for investors, we define the discount factor such that $\theta_{s+1}^S = \beta^S(\bar{C}_s^S)\theta_s^S$, with $\beta^S{}'(\bar{C}_s^I) \leq 0$, where $\bar{C}_s^I$ is the economy-wide aggregate consumption of savers. We make the assumption that savers are inherently more 'patient' than investors, in the sense that:

$$(10) \qquad \beta^S(x) > \beta^I(x) ,$$

for all feasible values of $x$. Assumption (10) ensures that savers will lend to investors, even in a steady state where the leverage constraint (3) is not binding.[3]

Savers purchase the fixed asset, and buy debt from investors. They receive wage income from working in the final goods sector, and returns on their lending to investors. In addition,

[3] An alternative, but considerably more difficult, approach to achieving an equilibrium with levered investment is to assume that investors are less risk averse than savers. Solving a model with leverage based on risk preferences would be substantially harder than the approach we follow, because we would need to solve a full stochastic model to a higher order of approximation.

they have a residual ‘home production function’ for use of the fixed asset. Thus an individual saver owning $k^S_{1,t}$ of the fixed asset produces $G(k^S_{1,t})$ in terms of home production, where $G'(k^S_{1,t}) < 0$. For simplicity, we assume that home production is perfectly substitutable with the final good in savers’ preferences. With this assumption, we may write the saver’s budget constraint as:

(11) $$C^S_t + q_{1t}k^S_{1t} = W^S_t + q_{1t}k^S_{1t-1} + G(k^S_{1t-1}) + B^S_t - R_{t-1}B^S_{t-1}.$$

Note that by assumption, savers purchase only the domestic fixed asset. They do not have access to the same investment opportunities as investors, and therefore only have use for the domestic fixed asset, which may be utilized in home production. On the other hand, savers’ purchases of debt from investors are unconstrained.

The first order conditions for the optimal choice of $k^S_{1t}$ and $B^S_t$ are simply:

(12) $$U'(C^S_t) = E_t\beta^S(C^S_t)U'(C^S_{t+1})\frac{(q_{1t+1} + G'(k^S_{1,t+1}))}{q_{1t}},$$

(13) $$U'(C^S_t) = E_t\beta^S(C^S_t)U'(C^S_{t+1})R_t.$$

**Production Firms**

Production firms in each country hire capital and fixed assets in order to produce. Firms are competitive, and maximize profits given the production function:

(14) $$Y_t = A_tF(L_t, K_t),$$

where $L_t$ is effective employment, and $K_t$ is the firm’s use of the fixed asset. We allow for labor supplied by investors and savers to have different fixed productive content. Thus $L_t = \eta^I L^I_t + \eta^S L^S_t$, where $\eta^I$ and $\eta^S$ are fixed effective productivity factors. Profit maximization then implies that:

(14) $W_t^I = \eta^I A_t F_1(L_t, K_t)$,

(15) $W_t^S = \eta^S A_t F_1(L_t, K_t)$,

(16) $R_{1K,t} = A_t F_2(L_t, K_t)$.

**Equilibrium**

An equilibrium of the two country world economy entails market clearing for the world market of the fixed asset as well as each country's debt market. Thus, for the home economy, it must be the case that:

(17) $nk_{1,t}^I + nk_{1,t}^{*I} + (1-n)k_{s,t}^S = 1$,

(18) $nB_t^I + (1-n)B_t^S = 0$,

where $k_{1,t}^{*I}$ represents foreign country investors' real holdings of the home asset at the beginning of time $t+1$. In addition, the world market clearing condition must be satisfied:

(19)
$$\begin{aligned} &n(C_t^I + C_t^{*I}) + (1-n)(C_t^S + C_t^{*S}) = A_t F(\eta^I n + \eta^S (1-n), n(k_{1,t}^I + k_{1,t}^{*I})) \\ &+ A_t^* F(\eta^{*I} n + \eta^{*S}(1-n), n(k_{2,t}^I + k_{2,t}^{*I})) + (1-n)(G(k_{1,t}^S) + G(k_{1,t}^{*S})). \end{aligned}$$

This condition incorporates the fact that the total labor supply of investors and savers is $n$ and $1-n$ respectively, and total use of the fixed factor by final goods firms is equal to total holdings by domestic and foreign investors. The full equilibrium is then described by equations (2)-(6) and (11)-(18) for both the home and foreign country, and the world market clearing condition (19). This gives 27 equations in the 26 variables $C_t^I, C_t^S, C_t^{*I}, C_t^{*S}$, $k_{1,t}^I$, $k_{2,t}^I$, $k_{1,t}^S, k_{1,t}^{*I}$, $k_{2,t}^{*I}$, $k_{1,t}^{*S}$, $B_t^I$, $B_t^S$, $B_t^{*I}$, $B_t^{*S}$, $q_{1,t}$, $q_{2,t}$, $R_t$, $R_t^*$, $\mu_t$, $\mu_t^*$, $W_t^I$, $W_t^S, W_t^{*I}$, $W_t^{*S}$, $R_{1K,t}$ and $R_{2K,t}$, with one equation redundant by Walras' law.

**Properties of the Steady State**

Before examining the dynamics of deleveraging within the model, we first discuss some properties of the non-stochastic steady state. This is particularly easy in the case of $\mu = \mu^* = 0$, which is when the leverage constraints do not bind. Then it follows from a combination of (4) and (6) with (12) and (13) that the fixed asset is allocated efficiently between the final good sector and home production. That is, for the home economy, we have:

(20) $\quad G'(k_1^S) = AF_2(L, n\hat{k}_1^I)$,

where $\hat{k}_1^I = k_1^I + k_1^{*I}$ represents the total quantity of the fixed asset used in the final goods production sector. Thus the fixed asset is allocated efficiently in the sense that its marginal product is equalized in home production and final goods production.

In combination with the resource constraint $n\hat{k}_1^I + (1-n)k_1^S = 1$, this uniquely determines the allocation of assets in final goods production. Therefore there is no interdependence across countries in asset allocation in steady state when leverage constraints do not bind. Hence output levels are independent across countries – a permanent increase in productivity $A$ affects home output but not foreign output.

In fact, we can extend this result further. In the case where the leverage constraint never binds, it is easy to see that there is no interaction between asset allocations across countries at all, at least up to a first order approximation. This can be seen by taking a linear approximation of (4), (6), (12) and (13), to obtain the condition: $\gamma_1 \frac{dk_{1,t+1}^S}{k_1^S} = E_t \frac{dA_{t+1}}{A} + \gamma_2 \frac{d\hat{k}_{1,t+1}^I}{\hat{k}_1^I}$, where $\gamma_1$ and $\gamma_2$ are constant coefficients. Hence the dynamic paths of asset allocations are independent across countries in the absence of balance sheet constraints.[4] Note that this holds despite the fact that, up to a first order, expected returns on all assets are equalized, both within and across countries.

When leverage constraints bind, we again use (4), (6), (12) and (13) to obtain the steady state condition:

$$G'(k_1^S) = \frac{\beta^I(1-\beta^S)}{\beta^S(1-\beta^I) - \kappa(\beta^S - \beta^I)} AF_2(L, n\hat{k}_1^I) . \tag{21}$$

From condition (3) it must be true that $\frac{\beta^I(1-\beta^S)}{\beta^S(1-\beta^I) - \kappa(\beta^S - \beta^I)} < 1$, so it follows that under binding leverage constraints, the final goods sector has an inefficiently low level of the fixed asset. More generally however, since discount factors are endogenous, the allocation of fixed assets across sectors will no longer be independent across countries. Asset allocation in the home country will depend on the level of productivity in the foreign country. Intuitively, this holds because, with free trade in equities across countries, returns to investors must be equal in both countries. Since returns interact with movements in consumption through the endogenous rate of time preference, (21) shows that the division of resources between home production and final goods must be linked across countries as well.

Even in the case of constant time preference, however, the presence of leverage constraints would still imply a dynamic interaction between output levels across countries that is absent without these constraints, because productivity shocks to one country will affect the tightness of leverage constraints across all financial markets. We explore this in detail below.

**Portfolio Choice**

[4] Of course this is not a robust feature, and would be altered in a model with endogenous labor supply or capital accumulation. But the main point here is to show that the presence of balance sheet constraints introduces substantial additional forces for cross-country correlations that would otherwise be absent.

We have already solved for the overall allocation of the fixed asset in each country in steady state, but not the ownership structure of equities. Thus while $\hat{k}_1^I$ is determined by (20) or (21) with non-binding or binding leverage constraints respectively, $k_1^I$ and $k_1^{*I}$ are not yet determined. Clearly, in order to analyze the dynamic response to productivity shocks in one country, it is necessary to understand the structure of equity holdings. To do this, we follow Devereux and Sutherland (2009) in using a 2nd order expansion of (8) to obtain an approximation of optimal portfolio holdings.

Since only investors have access to equity markets by assumption, it is sufficient to look at the portfolio decisions of home and foreign investors. To illustrate the application of Devereux and Sutherland (2009) to the present model, take equation (1), the budget constraint for home country investors. This may be rewritten as:

(22) $$C_t^I + NFA_t = W_t^I + R_{1Kt}\hat{k}_{1t-1}^I - q_{1t}(\hat{k}_{1t}^I - \hat{k}_{1t-1}^I) + r_{2t}NFA_{t-1} + r_{xt}\left[q_{1t-1}(k_{1t-1}^I - \hat{k}_{1t-1}^I)\right] + B_t - R_{t-1}B_{t-1}$$

where $NFA_t$ denotes net foreign assets, defined as $NFA_t = q_{2t}k_{2t}^I - q_{1t}(\hat{k}_{1t}^I - k_{1t}^I)$, and $r_{xt}$ is the *excess* return on the portfolio:

$$r_{xt} = r_{1t} - r_{2t} \equiv \left(\frac{q_{1t} + R_{1K,t}}{q_{1t-1}}\right) - \left(\frac{q_{2t} + R_{2K,t}}{q_{2t-1}}\right).$$

For given $NFA_t$, the portfolio choice may be described as the choice of $\alpha_t = q_{1t-1}(k_{1t-1}^I - \hat{k}_{1t-1}^I)$, which is the net holding of home country equity by home agents. If $\alpha_t < 0$, the investors diversify in the sense that less than 100 percent of all home equity is owned by home investors. Devereux and Sutherland (2009) show that, when the model is analyzed up to a first order approximation, $\alpha_t$ is a constant, and is determined by a combination of a second order approximation of (8) with a first order approximation of the rest of the model.

In the solution below, following Tille and Van Wincoop (2007), we extend (8) to allow for transactions costs of international financial trade that effectively limit international portfolio diversification. This represents a brute force technique for generating an equilibrium with home equity bias. In particular, we assume that an 'iceberg' cost factor given by $\exp(-\tau) \le 1$ reduces the returns that home investors receive from foreign investment, so that condition (8) becomes:

(8')
$$E_t U'(C_{t+1}^I)\left(\frac{(q_{1t+1}+R_{1Kt+1})}{q_{1t}} - \frac{(q_{2t+1}+R_{2Kt+1})}{q_{2t}}\exp(-\tau)\right) = 0 .$$

In addition, we follow Tille and Van Wincoop in assuming that $\tau$ is a small, second order term. This means that while it does affect the solution for the equilibrium portfolio, which is evaluated using a second order approximation of (8'), it does not impact on the first order dynamics of the model.

Note that given the revised definition of net foreign assets, the leverage constraint for home country investors becomes:

(23)
$$B_t \le \kappa(NFA_t + q_{1t}\hat{k}_{1t}^I) .$$

Thus, holding home asset prices constant, an increase in net foreign assets generated by either a current account surplus or a capital gain on the external portfolio will loosen the leverage constraint. But since $NFA_t + NFA_t^* = 0$, this will simultaneously tighten the leverage constraint facing foreign investors. Thus the degree to which leverage linkages govern the transmission of shocks across countries depends on the dynamics of net foreign assets, and these in turn are linked to portfolio choices made by home and foreign investors.

**Calibration**

Because the model is such a stripped down representation of a full-scale DSGE framework, lacking dynamics in labor supply, capital accumulation, and containing only a single world good, there are many dimensions in which the model's predictions will depart from reality. The aim of the exercise is solely to explore the way in which financial leverage constraints affect the cross country dynamics of asset prices, asset allocations, and levered investments, and to investigate the international transmission of 'deleveraging'. To do this, however, we need to choose parameter values for preferences, production technologies, and the leverage constraint itself.

Table 2 gives the set of parameter values used in the baseline model.

**Table 2: Calibration**

| Parameter | Value | Parameter | Value |
| --- | --- | --- | --- |
| $n$ | 0.5 | $\varepsilon$ | 0.5 |
| $\eta$ | 0.01 | $\omega$ | 0.36 |
| $\zeta$ | Discount factors 0.96 and 0.94. | $\sigma$ | 5 |
| $\kappa$ | 0.5, 0.8. | $\rho$ | 0.9 |

We assume that the measure of investors and savers is equal, so that $n = 0.5$. In the leverage constrained economy, this accords with the estimates of Campbell and Mankiw (1990) regarding the share of households that are subject to credit constraints in the US economy.

We assume a discount factor defined as:

$$\beta(C) = \zeta C^{-\eta}.$$

We set $\eta = 0.01$, and choose $\zeta$ so that in a steady state with binding leverage constraints, the lenders and borrowers have discount factors of 0.96 and 0.94 respectively. The parameter $\kappa$ directly determines the total value of new assets that investors can borrow. Since the model is calibrated in a symmetric way, net foreign assets are zero in steady state, so that investors' net worth, measured as total assets less debt, equals $q\hat{k}^I(1-\kappa)$.

Total leverage (investment relative to capital) is equal to $\frac{1}{1-\kappa}$. This leverage ratio has a significant affect on the model's dynamics. We examine two alternatives: First, we choose a relative low ratio of 2 ($\kappa = 0.5$), as in Bernanke et al. (1999). In response to the discussion of the importance of global deleveraging discussed above, however, and the high rates of leverage seen in the financial system in recent years, we also explore the implications of a higher value of $\kappa = 0.8$, corresponding to total leverage of 5.

We assume a Cobb Douglas final goods production technology, and let $F(L,K) = L^{\varepsilon}K^{1-\varepsilon}$. In order to have substantial propagation effects of credit constraints, Kiyotaki and Moore (1997) require that production in the borrowing sector is linear in the fixed asset. Kocherlakota (2000) shows that with a more conventional calibration allowing for decreasing returns, credit constraints have much less impact. We set $\varepsilon = 0.5$, implying substantial decreasing returns, yet find substantial effects of leverage constraints, as we will see. Our choice of $\varepsilon$ implies that fixed asset are slightly more important than conventional measures of capital's share in calibrations of the US economy.[5] In addition, initially we assume that effective labor productivity of savers and lenders is equal, so that $\eta^I = \eta^S = 1$.

Regarding the home production sector, we assume that $G = Z(k_1^S)^{\omega}$ and that the fixed asset is less important in this sector, so that $\omega = 0.36$, consistent with convention. We set $A = Z = 1$ in steady state. The combination of these assumptions imply that in steady state, 80 percent of the fixed asset is employed in final goods production. We follow the asset pricing

[5] For many emerging market economies, however, estimates of capital share equal to 50 percent are quite common.

literature (see, for example, Bansal and Yaron 2004) in setting a relatively high degree of risk aversion with $\sigma = 5$ in $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$. Lower values for $\sigma$ reduce the volatility of asset prices, but have little qualitative effect on the results otherwise.

We focus on shocks to the productivity of final goods in each country. The stochastic process for final goods productivity is modeled as:

(24) $$\log(A_t) = \rho \log(A_{t-1}) + \upsilon_t,$$

where $\rho = 0.9$, $E_{t-1}\upsilon_t = 0$, and $\sigma_\upsilon^2 = 0.02^2$. Foreign productivity is driven by the same process, and we assume that foreign and domestic productivity shocks are uncorrelated.

## 4. Deleveraging Effects of Productivity Shocks

### No leverage constraints

We first examine the impact of a 1 percent negative productivity shock in the home country, in the environment without leverage constraints. Figures 3 and 4 describe the impact of the shock on consumption of investors, asset prices, lending by savers, asset allocation, the internal lending rate, and the consumption of savers. Figure 3 represents the case where portfolio diversification is restricted by second order transactions costs as described above, while Figure 4 describes the case of unrestricted portfolios. In the unrestricted case, investors in the home country choose values for $k_1^I$ and $k_2^I$ to satisfy (8'), evaluated up to 2$^{nd}$ order, with $\tau = 0$. This involves home investors having a bias *against* home equities. Because the investors are exposed to non-diversifiable risk from wage income, which is positively correlated with the return on home equity, with an unrestricted portfolio they will hedge this risk by taking a larger position in foreign equity than home equity, as discussed in Baxter and Jehrmann (1997). Given the calibration of the model, in an unrestricted equilibrium, home investors would hold only 20 percent of total home equity (i.e. 20 percent of the fixed assets which are invested in the home final goods technology), with foreign investors holding the remaining 80 percent. Since this is clearly counterfactual, we use the iceberg cost variable $\tau$ as a crude mechanism to match the optimal portfolios more closely with observed home bias in equity holdings. In Figure 3 $\tau$ has been chosen so that $k_1^I = .75\hat{k}_1^I$, implying that home investors hold 75 percent of home equity. Figure 4, by contrast, illustrates the counterfactual case where there is full consumption risk sharing for investors due to unrestricted diversification, and $k_1^I = .2\hat{k}_1^I$.

Besides the implication for investors' consumption, the response to the productivity shock is quite similar in each of the Figures. Without leverage constraints, the impact of a fall in home country productivity is to reduce consumption of investors in both countries, by identical amounts in the case of unrestricted diversification. The shock represents a temporary fall in the consumption of investors in both countries. But since consumption is expected to increase in the future, real interest rates must rise. The combination of a persistently lower return on the home asset and rising real interest rates means that the home asset price must immediately fall.

Without leverage constraints, all returns are equalized, at least up to a first order approximation, for investors to be willing to hold all assets in their portfolios. Thus the price of foreign assets must also fall. That is, arbitrage implies that the rate of return to lenders rises by the same amount in both countries, even though lenders do not directly engage in international borrowing or lending. But the pattern of lending moves in completely different directions in the two countries, as do lenders' portfolios. In the home country, there is a fall in investment in the fixed asset in the final goods sector simply because this sector has suffered a persistent negative technology shock. This leads to an increase in the holdings of the fixed asset by lenders. They shift the composition of their portfolios from debt towards increased holdings of the fixed asset.

Thus lending falls in the home country. In the foreign country by contrast, there is no change at all in the allocation of the fixed asset. But lending in the foreign country actually rises, as investors borrow more from lenders in order to cushion against the temporary fall in their investment income.

A different way to see this is that in the foreign country, lenders are offered a higher rate of return on their lending, and are willing to purchase more debt from foreign investors. Either way we look at it, lending rises in the foreign country, while it falls in the home country. In this sense, there is no international transmission of de-leveraging.

The impact of the shock on lenders' consumption in the two countries also moves in opposite directions. Lenders in the home country lose, since they suffer a direct fall in their wage income. Lenders in the foreign country gain, since they lend more at higher interest rates, and their wage income and holdings of the fixed asset are unaffected. Clearly lenders cannot achieve full consumption risk-sharing, since they cannot directly hold a claim on the equity of the other country.

In the economy without leverage constraints then, the international transmission of shocks is limited, and clearly counterfactual, relative to the discussion of the empirical evidence of financial spillovers in section 2. A negative productivity shock in the home country leads to domestic 'de-leveraging', as investors reduce both their borrowing and holdings of fixed assets. But there is no foreign de-leveraging. Investment in fixed assets is completely unaffected in the foreign country, and foreign investors actually increase their borrowing. More critically, there is no international transmission of the shock in GDP at all. Since the foreign asset allocation is unaffected by the domestic shock, foreign output is unchanged. Thus, in the absence of credit market imperfections, the possibility for the international transmission of shocks through balance sheet deleveraging is limited.

**Leverage Constraints and International Transmission**

Figures 5-8 show the impact of a negative productivity shock in the home country in the model where leverage constraints bind in both countries. Figures 5 and 6 illustrate the case where the leverage ratio is 5, the former when investors' portfolios are only partly diversified due to the presence of transactions costs, and the latter with unrestricted portfolio diversification. Figures 7 and 8 illustrate the case of a lower leverage ratio of 2, in the case of partial and full portfolio diversification, respectively. In all cases there is a clear pattern of global deleveraging in response to the shock.

With high leverage and unrestricted portfolios (Figure 6), the home investor wishes to hold only 47 percent of total home equity. While there is still some 'foreign equity bias' here, it is far less than in the economy without leverage constraints. That is, in equilibrium without portfolio transactions costs, investors wish to hold more of their own equity when there are leverage constraints than when there are not. The reason is that (as we see below), the positive transmission of deleveraging across countries in the economy with binding leverage constraints will make equity returns more positively correlated. As a result, the gains from equity diversification are lessened. In Figure 5, we again calibrate $\tau$ so that investors hold 75 percent of domestic equity.

Without leverage constraints, the fall in home country productivity leads to a fall in asset prices in both countries, and a fall in investor's consumption. But now the fall in asset prices leads to a tightening of the leverage constraint, both in the home and foreign countries. This leads to a reduction in borrowing by investors in both countries, and a consequent reduction in investment in fixed assets. Note that for the foreign country there is no direct fall in the productivity of the domestic final goods sector.

In addition, the price of the foreign equity falls. But in spite of there being no *direct* shock to $R_{2K,t+1}$, and a fall in the price of the asset, there is still a fall in demand for the asset by investors in both countries. This is the essence of the 'inverted demand curve' for assets that characterizes episodes of de-leveraging, emphasized by Aiyagari and Gertler (1999). Here it is taking place as a spillover from one levered investor to another, as emphasized by Krugman (2008). That is, a fall in the price of the asset held by one investor leads to tightening of leverage constraints and a fall in demand for both the original asset and other assets held in the portfolio.

Even in the case where portfolios are only partly diversified, there is a very high correlation across countries in borrowing and investment. The decline in leverage is so great that the internal lending rate in each country immediately falls. Again, note that this is in response to a temporary shock, so that future consumption of investors is expected to increase. But because investors are subject to leverage constraints, the path of their consumption is de-linked from the path of interest rates. To see this more clearly, note from (6) that there is a conflict between the Fisherian determinants of real interest rates, and the effect of binding leverage constraints. Since consumption falls for both home and foreign investors, but is expected to rise in the future, real interest rates should rise. But this effect is more than offset by the increase in the shadow price of borrowing due to the leverage constraint. The fall in asset prices leads to such a tightening of leverage in both countries, and correspondingly a large rise in the shadow price of borrowing, that the real interest rate offered by lenders falls rather than rises.

The fall in asset prices is of a similar order of magnitude in the leveraged constrained economy as in the unconstrained economy. Asset prices display a V-shaped response, however, falling by less immediately than in the second period. This is due to the fall in lending rates. Since lenders are unconstrained, the fall in returns on lending must be accompanied by a fall in the expected returns on the lenders holding fixed assets for home production. Hence, immediately following the shock, asset prices are expected to fall further.

Note that there is a distinct difference between the constrained and unconstrained economy not just in the direction of international transmission of shocks, but also in the scale. In the unconstrained economy, a one percent decline in final goods sector productivity leads to an approximately one percent fall in borrowing from lenders, but only a 0.3 percent reduction in fixed asset investment. Not only is there an absence of transmission of international deleveraging transmission, but also the domestic impact of the shock is relatively mild. By contrast, the response of the constrained economy is larger by orders of magnitude. Borrowing falls by almost 5 percent in the home economy, and investment in fixed assets by almost 4 percent. Even in the foreign economy, the de-leveraging multiplier is very large – investment falls by over 3 percent and borrowing falls by 4 percent in the foreign economy. In both countries, the response to the shock is proportionally much larger than the shock itself, due to the interaction of asset price declines and binding leverage constraints.

Consumption of home and foreign investors also falls by more in the constrained economy than in the unconstrained economy, although the decline is less persistent with leverage constraints. Also, in contrast to the economy without constraints, consumption falls for all categories of households, both for investors and lenders in both countries. The fall in interest rates on lending in the foreign country eliminates the positive wealth effect we saw foreign lenders receive in the unconstrained economy.

When portfolio diversification is unrestricted, Figure 6 shows that the international transmission of the shocks is heightened even more. In fact, we get the surprising prediction that deleveraging and disinvestment is greater in the foreign country than in the home country! Borrowing falls by 5 percent in the foreign economy, and investment 4 percent, while the equivalent reductions in the home economy are 4 percent and 3.5 percent respectively.

Why does greater portfolio diversification magnify the international propagation effects of the shock? The reason is clear from (3), or (23). Increased diversification leads to a greater sensitivity of the foreign leverage constraint to the domestic asset price. A home country productivity shock leads to a greater fall in the home asset price than the foreign asset price. The greater is the exposure of the foreign portfolio to the home asset price, the greater is the negative transmission on leverage constraints following a negative shock to home productivity. In the completely unrestricted portfolio equilibrium (when $\tau = 0$), the foreign country is over-weighted in home equity, as part of an optimal risk sharing arrangement. So the macroeconomic reverberations of the shock are *greater* in the foreign country than the domestic country.

A second important feature of Figure 6 is that full portfolio diversification cannot ensure full consumption risk sharing in the leveraged constrained economy, as it did in the economy without constraints. This is because the evolution of domestic interest rates, and hence the domestic debt burden facing investors, is no longer identical across the two countries. Since equity market diversification cannot hedge away risk associated with country specific interest rate movements, unrestricted equity market integration (i.e. when $\tau = 0$) cannot achieve full consumption risk sharing in response to productivity shocks. Figure 6 shows that home and foreign investors' consumption are initially equalized, but foreign consumption rises above home consumption as the home lending rate is higher during the transition to a steady state.

Figures 7 and 8 illustrate the response to a home productivity decline in the case where leverage is lower ( $\kappa = 0.5$ ), when portfolio diversification is partial and full, respectively. In Figure 8, full diversification entails the home country investors holding only 25 percent of the home equity, while in Figure 7 home country investors are constrained to hold 75 percent of home equity. The pattern of responses is very similar to that in the high leverage case, but the magnitude of the responses is lower – borrowing and asset disinvestment are less than in the previous case, although there is still a substantial multiplier effect of the original shock on total leverage.

**Unconditional Moments**

Table 3 reports the unconditional moments of the model under the assumption that productivity shocks in both countries follow identical but uncorrelated distributions, given by (24).

**Table 3 Unconditional Moments**

| Leverage constraints | None | None | High | High | Low | Low |
|---|---|---|---|---|---|---|
| Diversification | Partial | Full | Partial | Full | Partial | Full |
| SDEV($C^I$) | 2.9 | 3.0 | 3.9 | 4.0 | 3.1 | 2.9 |
| SDEV($C^S$) | 0.4 | 0.5 | 0.8 | 0.9 | 0.5 | 0.5 |
| SDEV($q$) | 4.7 | 5.2 | 4.7 | 4.9 | 5.1 | 5.0 |
| SDEV($R$) | 0.6 | 0.7 | 2.0 | 2.0 | 0.9 | 0.9 |
| SDEV($\hat{k}^I$) | 0.8 | 0.9 | 11.0 | 11.3 | 4.2 | 4.0 |
| CORR($q_1, q_2$) | 0.98 | 0.99 | 0.99 | 0.99 | 0.99 | 0.98 |
| CORR($\hat{k}_1^I, \hat{k}_2^I$) | 0.02 | 0.04 | 0.99 | 0.98 | 0.99 | 0.98 |
| CORR($R, R^*$) | - | - | 0.99 | 0.99 | 0.99 | 0.97 |

As is evident from the Figures, the model with binding leverage constraints displays substantially more overall volatility than the model where leverage constraints are absent. Consumption of investors is 50 percent more volatile with high leverage constraints, and consumption of savers is twice as volatile. Asset price volatility is relatively unchanged with and without leverage constraints, but investment volatility is vastly higher, as is obvious from a comparison of Figures 3 and 5. Investment correlation across countries is essentially zero in the economy without leverage constraints. It is almost perfect in the economy with leverage constraints. Table 1 therefore underscores the main message of the paper. In the presence of credit market imperfections, balance sheet linkages across financial institutions can generate a very powerful mechanism for the international transmission of business cycle shocks.

## 5. Conclusions

To follow..

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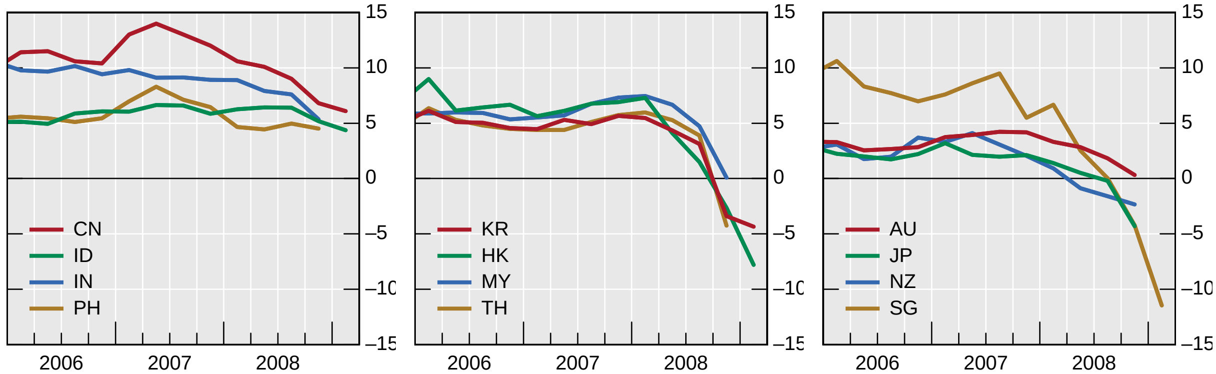
**Figure 1**

**Real GDP growth[1]**

In percent




[1] Year-over-year changes in real GDP. AU = Australia, CN = China, HK = Hong Kong, IN = India, ID = Indonesia, JP = Japan, KR = South Korea, MY = Malaysia, NZ = New Zealand, PH = Philippines, SG = Singapore, TH = Thailand.

Source: national data.

**Figure 2 US Inflows and Outflows, TIC data**
**Total**

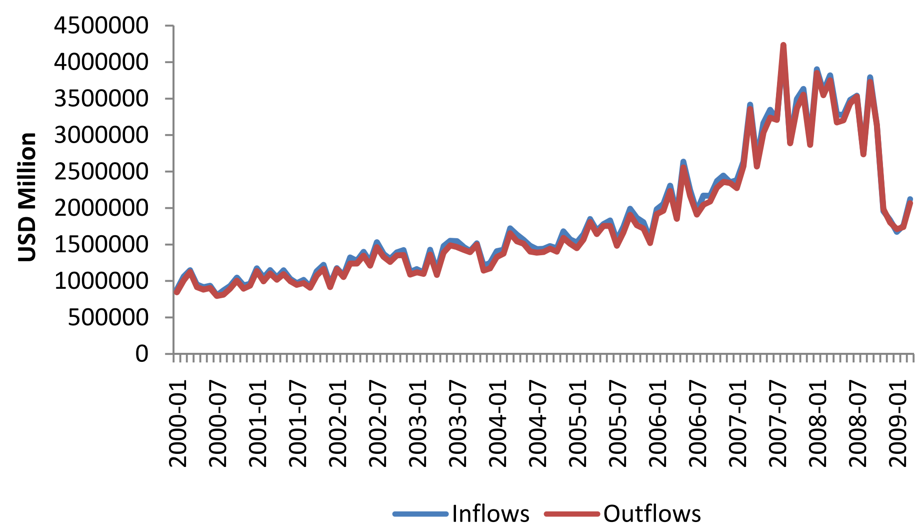


Figure 3: No leverage constraints, partial diversification

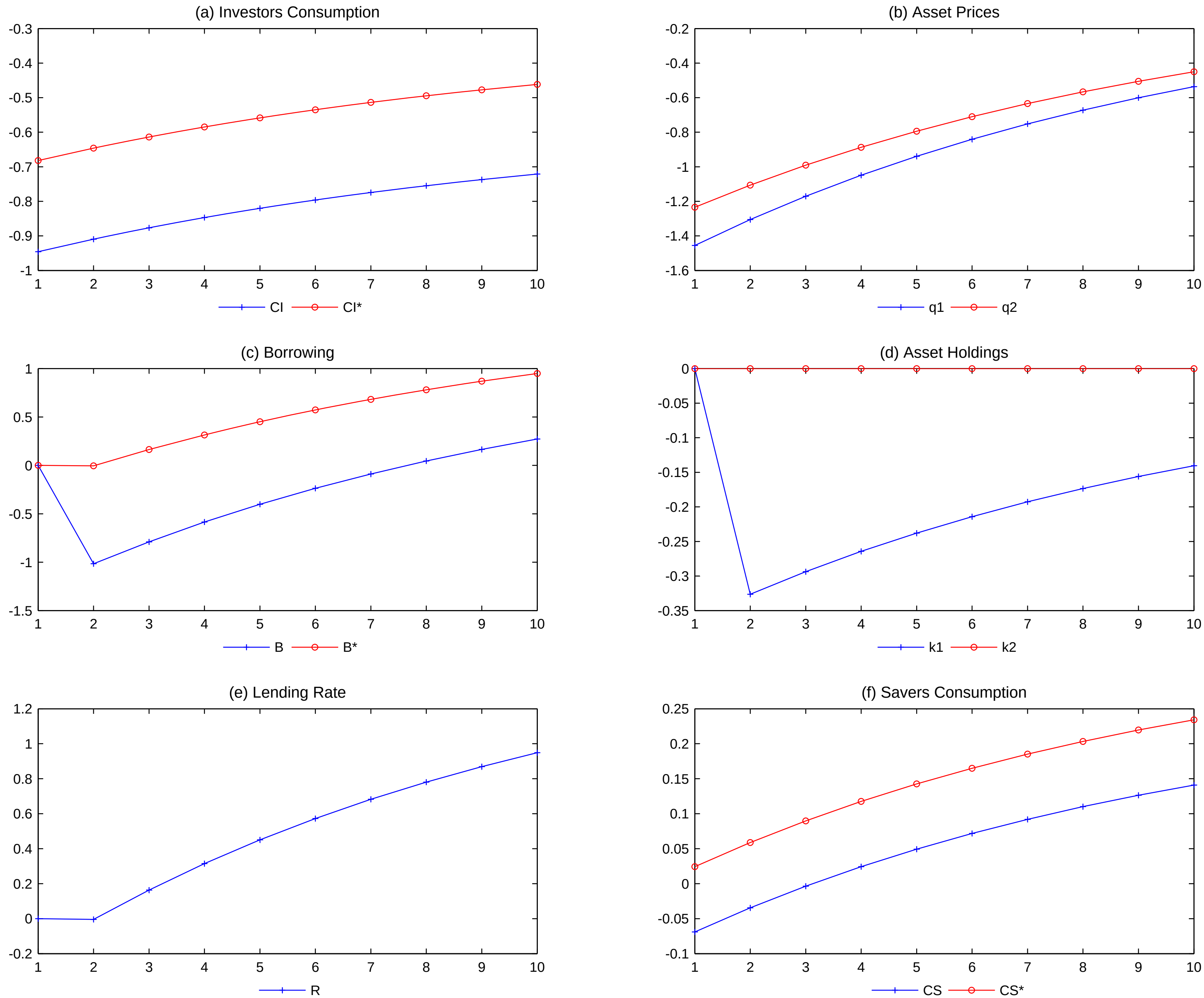


Figure 4: No leverage constraints, full diversification

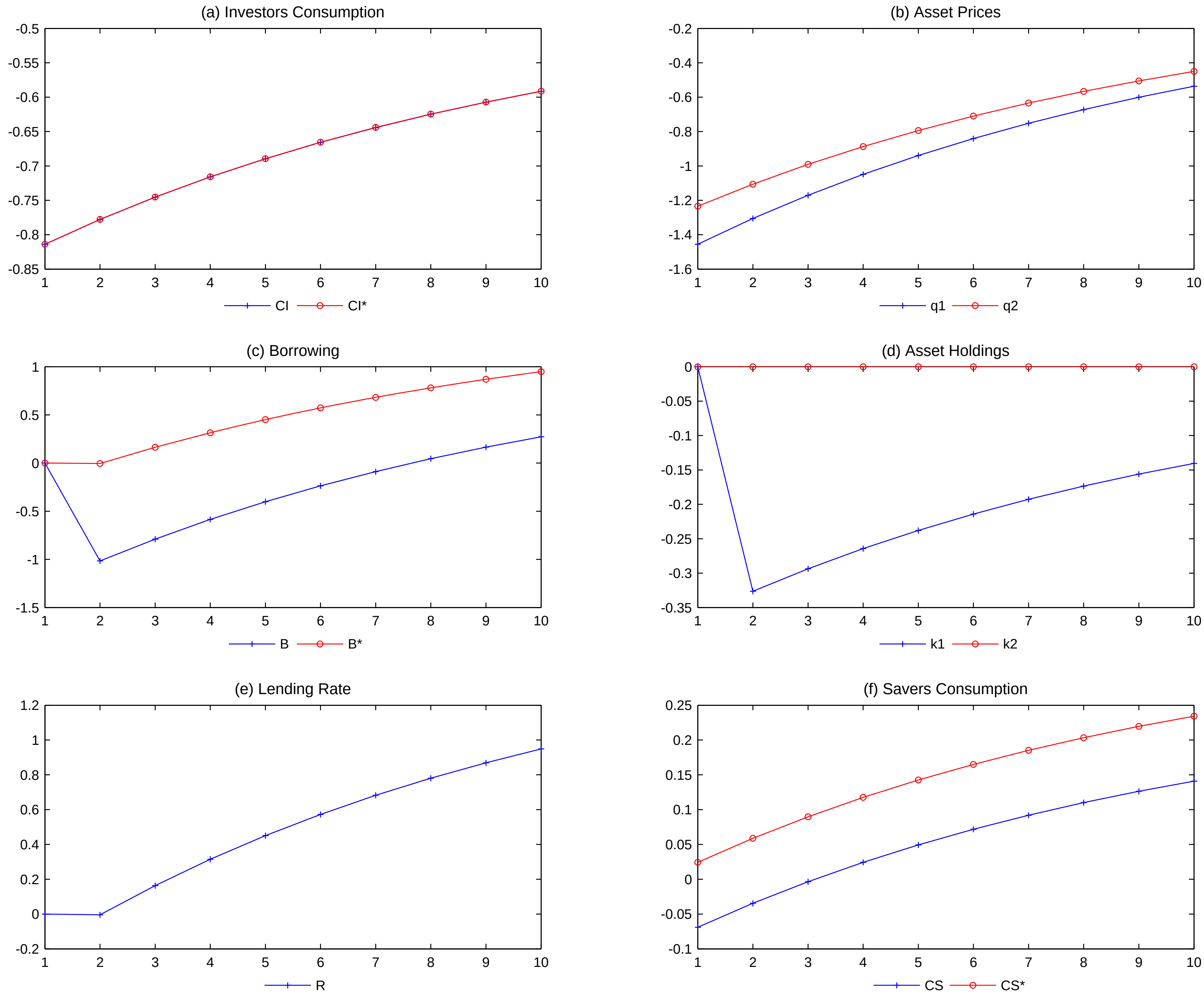


Figure 5: High leverage constraints, partial diversification

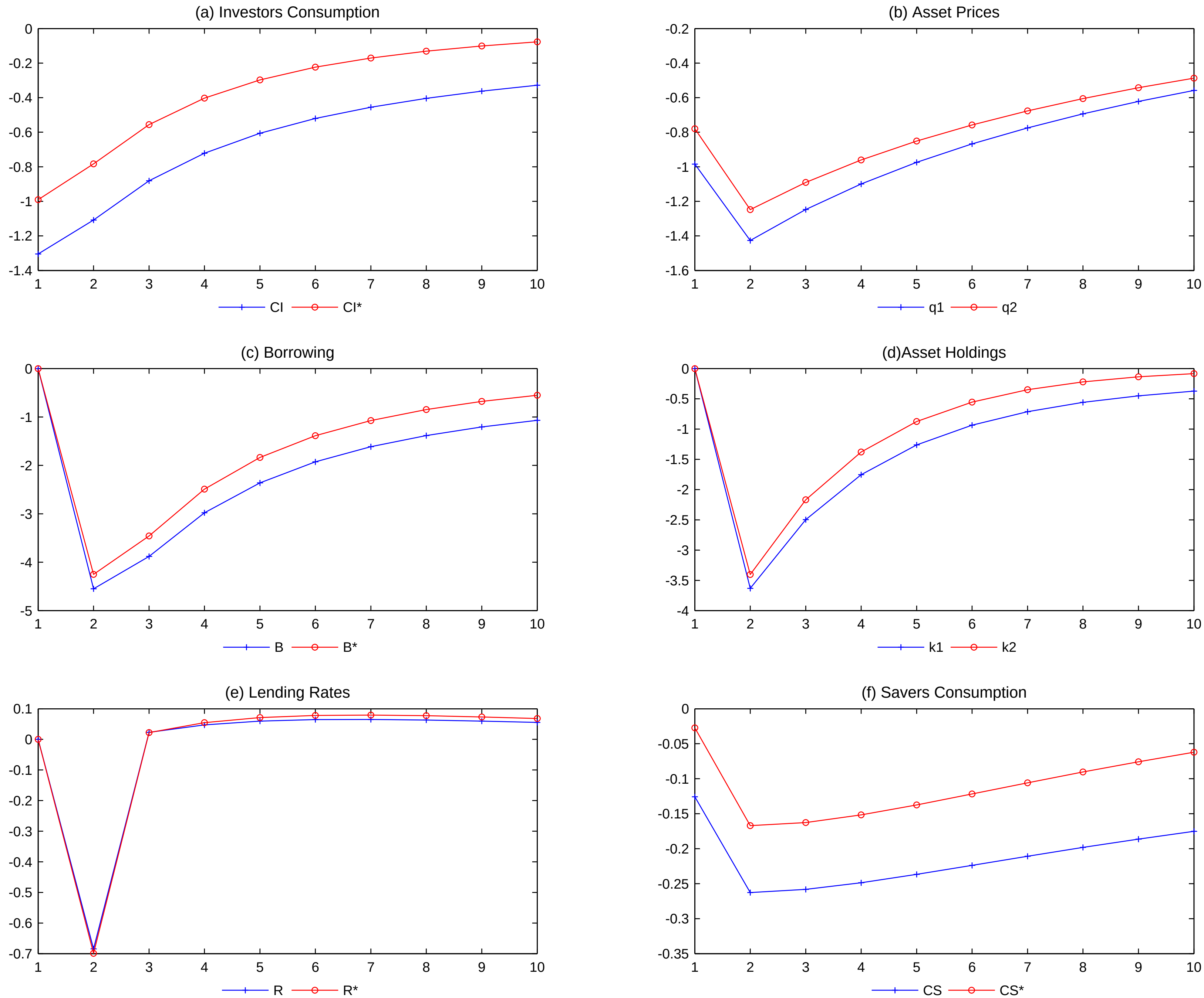


Figure 6: High leverage constraints, full diversification

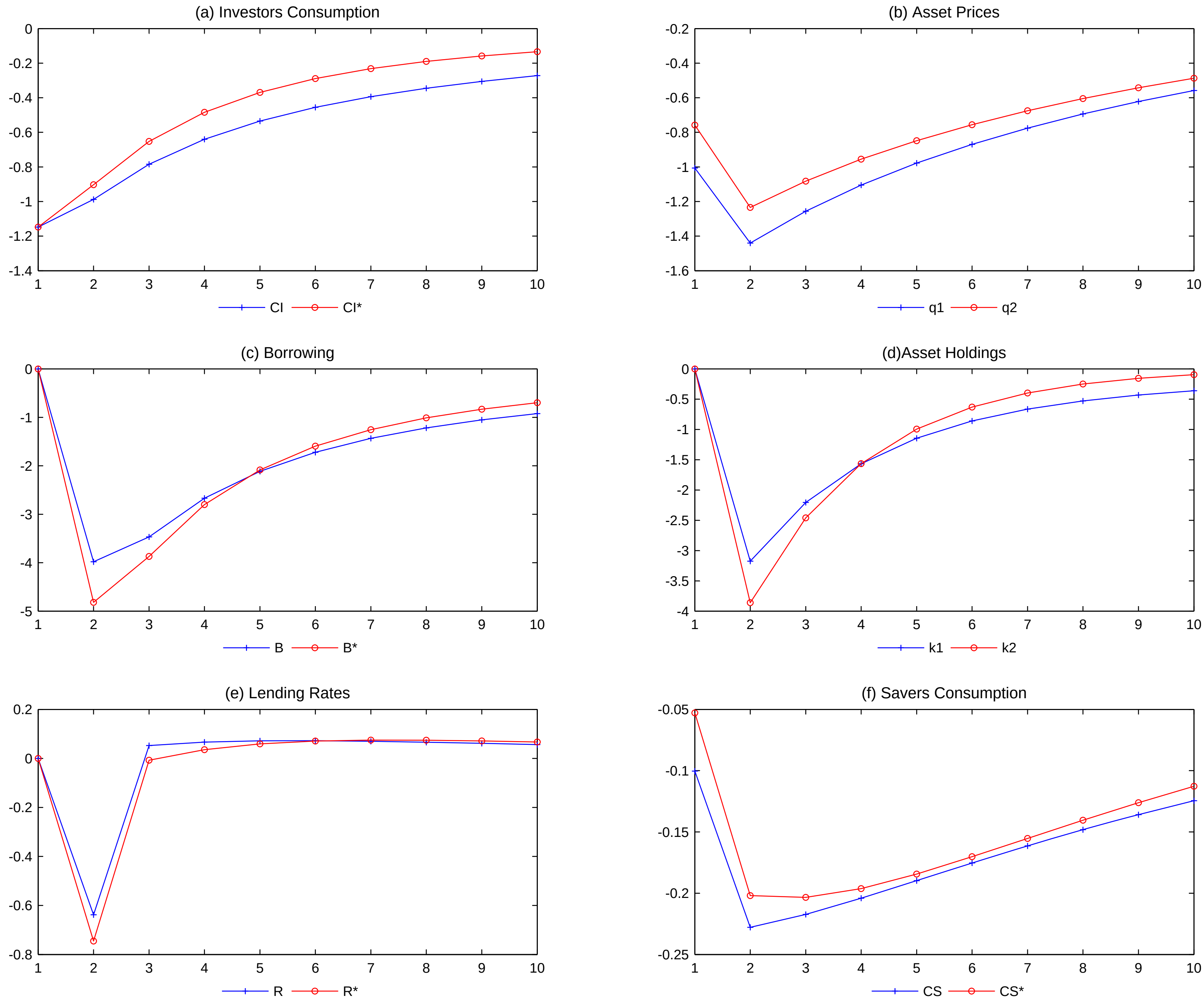


Figure 7: Low leverage constraints, partial diversification

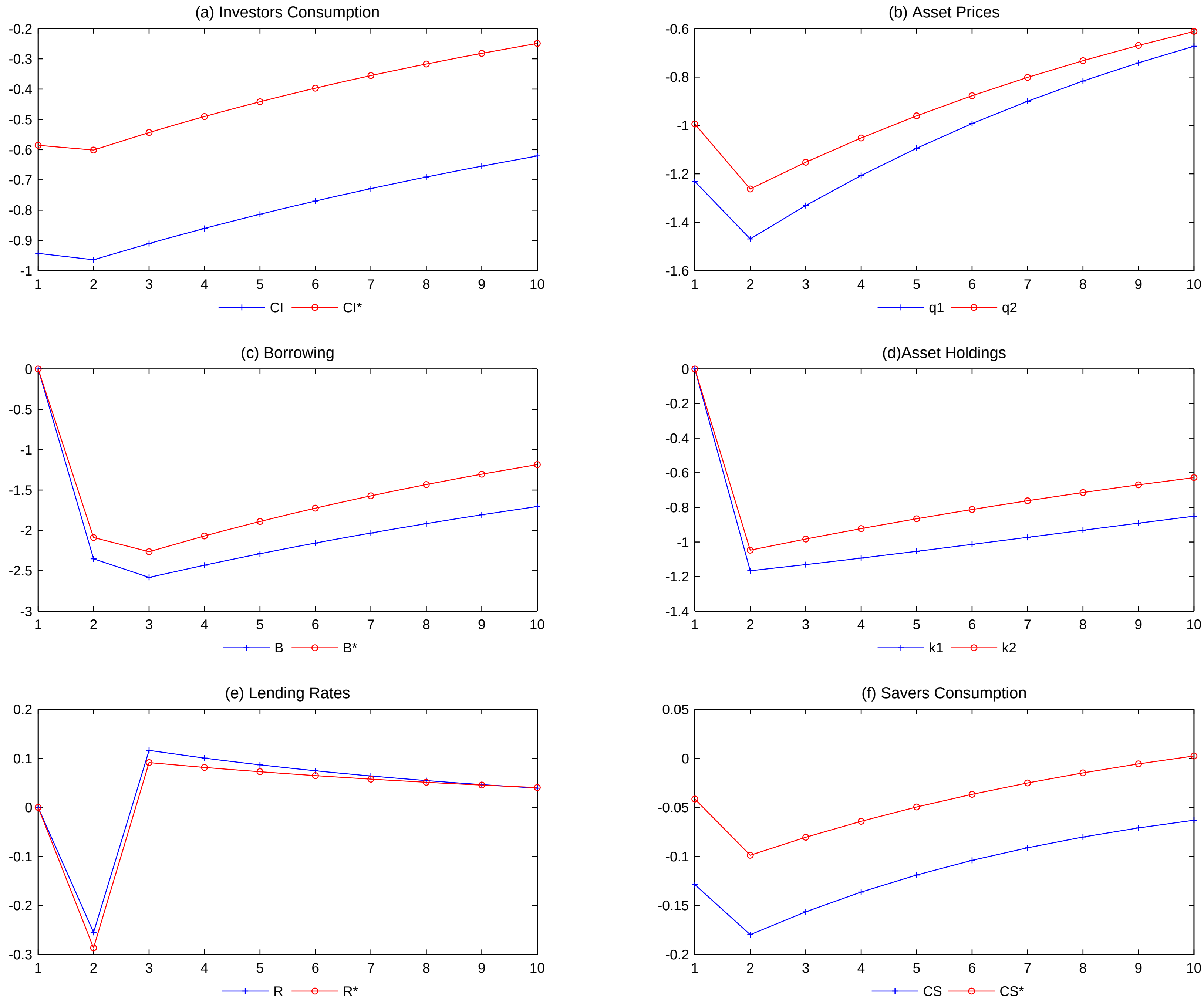


Figure 8: Low leverage constraints, full diversification