ON 'MONEY' IN ISLM AD/AS MODELS

T. K. Rymes and C. Rogers

Carleton University and Adelaide University

Working Paper 00-2

SCHOOL OF ECONOMICS
Adelaide University
SA 5005 AUSTRALIA
ON 'MONEY' IN ISLM AD/AS MODELS

T. K. Rymes and C. Rogers

Contact author:
Colin Rogers
School of Economics
Adelaide University SA 5005
AUSTRALIA
Tel: +61 8 8303 5528
Fax: +61 8 8223 1460
colin.rogers@adelaide.edu.au
"On ‘Money’ in ISLM and AD/AS Models"¹

Introduction

Hicks's ISLM model interpretation of Keynes's theory² is subject to much controversy.³ In this paper, we focus upon the 'real balance' effect and its role in ISLM and AD/AS analyses.⁴ We shall argue that ISLM and AD/AS require 'nominal anchors'. We live in a world where, increasingly, the 'money' in the ISLM and AD/AS model no longer exists (as Keynes imperfectly understood in his TREATISE ON MONEY). There are no longer any nominal anchors, rather they have been replaced by discretionary policy. As Professor Ferris nicely sums up our position, we argue that central banks are fundamentals.

A Review

In any standard ISLM analysis, the IS schedule is

\[ y = \varphi (i, y, K + \overline{M}/P) \]  

(1)

where \( y \) is output in 'real' terms or constant prices, \( i \) is (a complex of) the 'real' rate(s) of interest, \( K \) is the given 'real' stock of capital, \( \overline{M} \) is a given nominal stock of money, such as circulating currency issued by the central bank and the nominal stock of (say, demand) deposits held by the nonbank public as liabilities of the banks. While the expected rate of change of


---

¹ Despite the greatly appreciated comments of Professors Steve Ferris and Nicholas Rowe and general remarks by Professor Jack Galbraith, all error and confusion remain ours.


prices is taken to be zero in our discussion, it can be easily adapted to expectations of inflation or deflation. One can rewrite (1.1) as

$$y = \varphi(i, y, \overline{K} + \overline{H}/P)$$

(2)

where \(\overline{H}\) is a given nominal stock of fiat, base or outside money made up of circulating currency and the nominal stock of reserves held by the banks, both liabilities of the central bank.\(^5\)

It is assumed that \(\varphi_1 < 0, 0 < \varphi_2 < 1\) and crucially \(\varphi_3 > 0\).

The LM schedule is

$$\frac{\overline{H}}{P} = \mu(i, y, \overline{K} + \overline{H}/P)$$

(3)

where \(\mu_1 < 0, \mu_2 > 0, 1 > \mu_3 > 0\).

The two schedules determine \(i\) and \(y\) for any given \(P\). A lower \(P\) will be associated with a higher \(y\) and a higher/lower \(i\) (see appendix).\(^6\) The negative relationship between \(P\) and \(y\) gives the AD curve, shown (in the appendix) convex in \((P, y)\).

The relationship between ISLM and AD/AS, when the (money wage rate and) price level is lower at \(P^*\) than at \(P\), is illustrated in Figure 1.

---

\(^5\) One does this by means of the standard well known formula

$$M = H \frac{1 + \sigma}{C + \sigma},$$

implying that \(H\) is considered as part of the wealth of the private sector. Though it is a liability of the government sector, it is assumed in standard analysis that changes in its real value do not affect the expenditure of the public sector. Our paper argues that this formula no longer has any content in modern monetary economies.

\(^6\) By means of writing \(y\) as a function of \(L\) (and \(K\)) and the price level being equal to the money wage rate divided by the marginal physical product of labour, the whole analysis can be cast in terms of interest rates and levels of employment, with the levels of employment being inversely related to the level of money wage rates. Such a presentation indicates clearly how different such analyses are from that of Keynes.
It is well known that one is dealing with lower price levels not with falling price levels, the latter perhaps being associated with lower aggregate demand.\footnote{For different conclusions, see J. Tobin, "Price flexibility and output stability: an old Keynesian view", \textit{Journal of Economic Perspectives}, VII, 1993, 45-65 and M. Friedman, \textit{PRICE THEORY} (1976), 319-321. Tobin argues that falling prices will create instability because of exacerbated Fisherian debt-deflation effects, excess demand for money because nominal interest rates cannot go negative and consequently even greater falls in prices, in short, price and output instability. Friedman distinguishes between output and real private income, out of which expenditure on output takes place. Private real income is output plus (minus) real capital gains (losses) from holding outside fiat money, with debt-deflation effects offsettingly distributional. The faster prices fall the greater is real private income and the greater are}
Ignoring negative interest rates, the so-called Patinkin ‘real balance’ effect (and the so-called Keynes effect, which is really just part of the Patinkin effect) means there is necessarily a level of prices (and money wage rates) such that y is at full employment, contrary to Keynes’s basic position that such a level may, not must, exist. One quotation is enough to represent a widely held view.

"But in [his] admirable attempt at formal theory, Keynes failed. His top-priority goal of articulating a model with an unemployment equilibrium... founndered on the Pigou-Patinkin real balance effect."1

**Initial Criticism**

Keynes knew of the Pigou effect and dismissed it as empirically unimportant.2 So do Friedman and McCallum.3 Here is a curious argument. While it is stated that one would not rely upon ‘real balance’ effects in the real world, they nevertheless prove that Keynes’s contention that there may exist a less than full employment equilibrium is without theoretical foundation. As Friedman (321) says,

"The Pigovian and [Friedman’s] the more far-reaching answer to Keynes’s proposition have been extremely important on a theoretical level in assuring that there is no basic flaw in our theoretical analysis. But I hasten to add that in my

---

expenditures on output. In the extreme case of no capital formation, real private income, by Friedman’s dynamic version of the real balance effect, would be greater the greater the fall in prices and real consumption expenditure would approach the full employment level of output. At that point prices and money wages would stop falling and Tobin’s spectre of instability set aside.


10 Friedman, *op. cit.* and Bennett T. McCallum, *MONETARY ECONOMICS: Theory and Policy* (1989). McCallum’s views about the empirical unimportance of the real balance effect are drawn up in the context of evaluating the welfare costs of steady inflation. The basic conclusion in his summary analytical framework, which must follow from the assumed presence of nominal anchors and the belief that policy is concerned with the manipulation of such costless fiat creatures, is that most macroeconomists accept as a *theoretical principle* that deviations of output or employment "...cannot be affected permanently by monetary policy."(216).
opinion neither corresponds to effects that are empirically important in the kind of economic fluctuations that actual economies experience." (our italics)

One of our objectives in writing this paper is to eliminate the mental tensions such extraordinary statements must create.\footnote{They are examples of what Coates calls scholasticism. See John Coates, THE CLAIMS OF COMMON SENSE: Moore, Wittgenstein, Keynes and the social sciences (Cambridge: Cambridge University Press, 1996)}

One can also say that, since $H$ includes circulating currency, which modern central banks do not control but rather allow to expand or contract to meet changes in the nominal (and real) demand for circulating currency, the partial endogenization of money undercuts the empirical and theoretical rational for the role of 'real balances' in the IS-LM model and the implication that differences in the price level can have equilibrating properties in the AD/AS models. While it is undoubtedly true that no modern monetary authority operates on the stock of circulating central bank notes or fiat coin to control price levels, they still have, according to the traditional position, the stock of reserves in the form of deposits held by private banks with the central banks on which to operate. While it can be argued that private agents, in a world of lower price levels, will use their holdings of circulating currency to pay off overdrafts and loans so that deposits and currency, $M$, in the hands of the public and price levels $P$, will be positively correlated, there need exist no such positive correlation between reserves of banks with the central bank and price levels. That is, while it might be argued that $M = M(P)$ such that $M_P > 0$ so that lower prices are associated with lower 'money' supplies, one cannot argue that $H = H(P)$ and $H_P > 0$ with similar conviction.\footnote{See Marc Lavoie, FOUNDATIONS OF POST-KEYNESIAN ECONOMIC ANALYSIS (Aldershot: Edward Elgar, 1992), See especially Chap 4 Credit and Money, Basil J. Moore, HORIZONTALISTS AND VERTICALISTS: The macroeconomics of credit money and Charles Goodhart, "Money supply control: Base or interest rates?", eds. K. Hoover and S. Sheffrin, MONETARISM AND THE METHODOLOGY OF ECONOMICS (Aldershot: Edward Elgar, 1995) reprinted in Goodhart, THE CENTRAL BANK AND THE FINANCIAL SYSTEM (Cambridge, MIT Press, 1995).}

\subsection*{Major Criticism}

The major point of this paper is that the stock of nominal fiat high powered money has vanished in modern monetary economies. There is no nominal anchor! There is no $H$ which will be a co-determinant of $P$ nor is there an $H$ which will ensure, via the 'real balance' effect, a full
employment price level. Keynes’s conjecture that there may exist less than full employment monetary equilibria remains correct.

It is now increasingly established that central banks exercise control over nominal and real magnitudes, in regimes where banks desire to hold zero reserves, not by altering the stock of reserves nor by fixing interest rates but rather by operating upon the spreads or the relative price of banking services. Central banks always affect the price of banking services which are essentially the supply of liquid, and accessible-at-least-cost, intertemporal transactions services. Central banks are always operating upon this real relative price so that the set of all relative prices cannot be ascertained independently of the activity of the monetary authorities. Just as Keynes argued we now must work with a monetary theory of value.

Modern central banks, such as the Bank of Canada and the Reserve Bank of Australia, have learned that by ‘setting spreads’, i.e. by charging higher than market interest rates on banks’ negative settlement balances and paying lower than market rates on banks’ positive settlement balances, they enforce banks to pursue overdraft and deposit policies such that the banks expect to be in zero settlement balances position in their clearings with the central Bank (hereafter just the Bank), unless the Bank is taking non-neutral action. Unless the Bank takes what we shall call enforcement actions, such as open market operations or repos or redeposit and drawdown of government deposits (or overdrafts) with the banks, the banking system, i.e. the set of all financial intermediaries offering transactions services, will always be in balance, in an overall zero settlement balance position with the Bank. Set aside circulating currency for a moment. In a world of paper transactions or increasingly electronic debits and credits, one individual’s debit (an increase in her overdraft or decrease in her deposit) must be matched by another individual’s credit (an increase in his deposit or decrease in his overdraft). If the individuals transact through different banks, her bank will be experiencing a negative settlement balance with the Bank while his bank will be recording an equivalent positive settlement balance. A market in settlement balances will exist, such that through their private clearing arrangements, a House, her bank will borrow his bank’s positive clearing balance at a rate lower than the Bank would charge while his bank would loan its positive settlement balance at a rate higher than could earned with the Bank. Hence, unless the Bank puts the banking system as a whole into non-zero settlement balance positions, the interbank market (or House) in settlement balances will clear at spreads or ‘service prices’ below those of the Bank.

Private non bank agents transact with banks and face interest rates an overdrafts which exceed rates paid by the banks on deposits, the spread said to be the ‘price’ of banking services. Of

---

course, the 'spread' the banks charge their customers is greater than that charged by the banks’ House and the 'spread' charged by the Bank.

From Baltensberger\(^\text{14}\), the liquidity costs of a competitive bank are given as

\[ L = iR + \int_R^\infty \rho (X-R) f(X) \, dX \]  \hspace{1cm} (4)

where \( i \) is the opportunity cost of reserves, \( R \) is precautionary reserves, \( \rho \) is the penalty rate charged (say) by the Bank for being in reserve deficiency, \( X \) is the net loss experience, \( f(X) \) the density function of such withdrawals assumed normal so that \( E(X) = 0 \). The liquidity costs are minimized when

\[ 1 - \rho \int_R^\infty f(X) \, dX = 0 \]  \hspace{1cm} (5)

so that if \( R \) is zero, \( \rho = 2i \), or what emerges is the '2 for 1' rule currently imposed by the Bank of Canada.\(^\text{15}\) That is, in order for a bank to conduct its overdraft and deposit policies such that it expects to be in a zero settlement balance position in the clearings, the penalty rate must be twice the opportunity cost rate, for example, the rate banks could earn on demand overdrafts.

The structure of interest rates implied by the '2 for 1' rule is that the Bank would pay a zero rate on positive settlement balances, \( i_p = 0 \). This implies that the cost of holding a positive settlement balance with the Bank is \( i = i_p = \delta_{cb} \) and that the penalty rate, designated \( i_m (=2i) \), implies that the cost of holding a negative settlement balances with the bank is \( i_m - i = \delta_{cb} \). The expression \( \delta_{cb} \) is the carrying cost, spread or service charge the Bank is levying for the provision of clearing and liquidity services.


When the Bank raises \( i_p \) without altering \( i_o \), it raises the charge it makes for the provision of services to the banks. The service charge is composed of two parts: the trivial debit and credit fees which 'price' the number of transactions; and the carrying cost, the 'price' for the provision of liquidity serves to them. Similarly, the House charges its members fees for its final clearing, as the banks swap settlement balances and for the monitoring costs involved in the provision of the liquidity services the House provides its member banks.

Individuals use the transactions services of banks since they (i) delegate the monitoring of persons to whom they would lend and from whom they would borrow to the banks because it is less costly for the banks, specializing in this activity, to do so; (ii) they use overdrafts and deposits for the execution of transactions (and may use private bank notes in addition) because, unlike other financial intermediaries, banks promise to keep unchanged the nominal value of such overdrafts and deposits \(^{16}\) and (iii) subject to known costs (the debit and credit fees and carrying costs) they debit and credit their deposits and overdrafts because of immediate of access on the part of depositors and non-immediacy of call experienced by overdrafters. \(^{17}\) Banks provide monitoring, a common service of financial intermediaries, but their assets and liabilities used in transactions are fixed in nominal terms with clear terms of accessibility and established interest rates, debit-credit fees and carrying costs.

A bank is able to offer immediacy of access to demand depositors and non-callability to overdrafters because it can tap the provisions of liquidity by the House. A bank will not necessarily demand payment of an overdue overdraft if it thinks the overdrafter's credit is good but is merely in an illiquid position if it can in turn borrow, at a cost in the House, in turn not subject to immediacy of call. Thus, the bank's overdrafters and depositors are using indirectly the immediacy of accessibility at least-cost services supplied by the House. Ultimately, of course, the banks and their House fall back on the services of liquidity, provided at cost, by the Bank.

Banks, though increasingly less so, charge for their services through 'spreads'. A simple revenue statement for a bank, with respect to overdrafts and deposits would be

\[
i_o - i_D
\]


\(^{17}\) A demand deposit has immediacy of access, a time deposit has an option which inhibits immediacy of access (or makes it more costly), with a time overdraft a bank cannot call so the overdrafter is not subject to harassment while a demand overdraft is subject to call on demand.
for a House

\[ i_B^sB - i_L^sL; \]

and for the Bank

\[ i_{cB}Dr - i_{cC}Cr. \]

where \(O\) and \(D\) are the bank's overdrafts and deposits, with their rates, \(i_o\) and \(i_d\); \(B\) and \(L\) are the borrowings from and lendings to the House by its members, with their rates \(i_B^s\) and \(i_L^s\); and \(C\), and \(D\), are the negative and positive settlement balances of banks with the central Bank, not swapped in the House, with their respective rates \(i_{cD}\) and \(i_{cC}\). We observe, in general, for demand overdrafts and deposits,

\[ i_{cD} > i_B > i_o > i_{cC} > i_L > i_{cC}. \]

One argues that \(i_{cB} - i_B\) is the service charge levied on overdrafters and depositors by banks,\(^{18}\) \(i_B - i_d\) is that levied by Houses on their members while \(i_{cD} - i_{cC}\) is the service charge levied by Banks on banks unable to obtain accommodation through the House, that is, the price charged for the ultimate liquidity provided by them.

Suppose the 'spreads' are such that the House and banks are producing a volume of nominal overdrafts and deposits - that is, are producing a volume of services - with which the Bank is content. Suppose the banks begin an expansion of overdrafts and deposits with which the Bank is not in accord. The expansion in banking output is too great. Modern Banks raise their spreads, i.e. they raise \(i_B\), keeping (say) \(i_{cD}\) unchanged. The Bank's spread is higher, and the House and clearing banks know the price of the liquidity services offered by the Bank is higher. To the House and clearing banks, the cost of straying from zero to negative settlement balances, compared with straying from zero to positive settlement balances, has risen.\(^{19}\) Competitive banks (and their House) will therefore try to unbalance their portfolios away from overdrafts towards deposits which of course, they cannot do. Their spreads, their service charges, will rise. Non banks will find therefore

\(^{18}\) The UN (United Nations, **SYSTEM OF NATIONAL ACCOUNTS 1993**) argues there exists a 'pure' rate \(i\), such that \(i - i_d\) is the service charge paid by overdrafters and \(i - i_B\) is that service charge paid by depositors.

\(^{19}\) In Canada, the Bank raises its borrowing and lending rates together, operating with 'bands'. The increases in its lending rate, the rate it charges for negative settlement balances, constrains banks to reduce their overdrafts. The increase in the borrowing rate, the rate the Bank pays on positive settlement balances, constrains banks to increase their deposits. Both represent an attempt by the banks to move towards positive settlement balances but, try as they may, unless the Bank accommodates, they cannot do so. Rates on overdrafts and deposits rise to the levels the Bank desires.
intertemporal surplus compared with deficit positions less costly and planned investment will contract relative to savings.

The Bank has not reduced reserves (there are none to reduce) by neither open market operations nor drawdowns, nor has it raised interest rates per se. Rather it has raised its spread\textsuperscript{29} and the House and its banks follow suit. The whole set of intertemporal transactions services bank provide has increased in relative price and the volume of bank output contracts.

Since banking output is used by firms and households as inputs into final demand components and as intermediate inputs in production the decline in banking output is associated with a contraction in final demand and the net output of the whole economic system.

The Bank attempts to thwart the expansion in banking, and aggregate output, not by contracting the growth of some particular definition of money or high-powered money nor by raising interest rates. Rather it does it by raising the price of liquidity - the charge for the service the Bank ultimately provides. Since the demand for liquidity provided by the banks, the House and the Bank is continually changing in a Keynesian world of uncertainty, the Bank must always be operating with the relative price of banking services and output. \textit{Central Banks therefore have unrelenting real effects in the economy.} Earlier views which had Banks changing some costless stock of fiat money can easily be shown, with model-consistent expectations, to ensure that Banks have no 'real' effects. Such views are, however, no longer applicable. Similarly, those views which had central Banks setting interest rates and able somehow to affect not just nominal but real interest rates seem questionable. No doubt by encouraging banking expansions which could be associated with higher growth rates, Banks might raise real rates of return, given positive links between growth rates and real rates of return, but the relationship between central Banking, charges in banking output and real rates of growth requires more theoretical and empirical study.

\textsuperscript{29} If banks, the House and the central Bank reduced their spreads and substituted instead specific carrying costs (or service charges) on overdrafts and deposits on borrowing and lending and on negative and positive settlement balances, then it would be clear that the Bank exercises control by raising or lowering its spread and not raising or lowering an interest rate (such as Bank rate).
Reconstruction of ISLM and AD/AS Analysis

Consider a simplified set of balance sheets in a banking equilibrium.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Government of Canada</strong></td>
<td></td>
</tr>
<tr>
<td>Deposits at Bank of Canada</td>
<td>Bonds held by Bank of Canada</td>
</tr>
<tr>
<td>Deposits at banks</td>
<td>Other Bonds</td>
</tr>
<tr>
<td><strong>Bank of Canada</strong></td>
<td></td>
</tr>
<tr>
<td>Bonds of the Government of Canada</td>
<td>Government of Canada deposits</td>
</tr>
<tr>
<td>Negative settlement balances</td>
<td>Positive settlement balances</td>
</tr>
<tr>
<td><strong>Banks</strong></td>
<td></td>
</tr>
<tr>
<td>Positive settlement balances</td>
<td>Negative settlement balances</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>Deposits (including Government of Canada)</td>
</tr>
<tr>
<td>Overdrafts</td>
<td></td>
</tr>
<tr>
<td>P_K Bank capital</td>
<td></td>
</tr>
<tr>
<td><strong>Non-Banks (Households and Firms)</strong></td>
<td></td>
</tr>
<tr>
<td>Deposits</td>
<td>Overdrafts</td>
</tr>
<tr>
<td>P_K (Non Bank capital)</td>
<td>Net Worth</td>
</tr>
</tbody>
</table>

It is to be noted that no nominal anchor exists. In nominal terms all magnitudes could be raised or lowered without any 'real' effects. It is only the liquidity services of the Bank which pin down nominal (and real) magnitudes in the economy. There is no H doing this. This being the case, the level of money wage rates and prices are indeterminate without Bank activity.
In terms of AD analysis, one starts with Figure 2.

Figure 2

The position of the AD curve is a function of real variables including the provision of liquidity services by the Bank. The lower its service charge the higher will be AD for any given level of prices. For a given level of money wages and Bank services charges we have a given AS curve. The gross output of the economic system is \( Q = Q[L, K, (O + D)] \) where \( Q \) is gross output, \( L \) and \( K \) are non-banking labour and capital and \( (O + D) \) is the output of the banking sector. However, banking output, \( (O + D) = F[L_p, K_p, (D_p + C_p)] \), is a function of labour and capital in banking and the services supplied by the Bank. Thus net output is \( Q = Q[L, K, (D_p + C_p)] \). In a Keynesian short run, costs are \( W_L + \delta_{CB} (D_p + C_p) \) where \( \delta_{CB} \) is the central Bank spread. For given \( W \) and \( \delta_{CB} \), the marginal cost or supply price of net output rises in the short run because of diminishing marginal products of labour and banking services as inputs. For a decrease in \( \delta_{CB} \), aggregate demand will increase and also the aggregate supply will shift to the right subject to diminishing returns to the provision of liquidity services.
See Figure 3 for an illustration of this case.

**Figure 3**

Full employment levels of output, \( y^* \), cannot be defined independently of the level of banking output. Since banking output cannot be defined independently of the flow of liquidity services from the Bank, no precise definition of full employment level of output is possible. There is thus no precise natural level of output because there is no natural level of outputs of banks, Houses and Banks.
We contrast our AD/AS analysis with those of new classical and new Keynesian economics in Figure 4.

**Figure 4**

In new classical economics, AS is given at the natural level of output, the AD is drawn to reflect the existence of ‘real balance’ effects. A reduction in AD by monetary contraction from $\text{AD}_0$ to $\text{AD}_1$ results only in lower prices with equilibrium at full employment assured by ‘real balance’ effects. See left panel of Figure 4. In new Keynesian economics, with sticky money wages and prices, a reduction in AD by monetary contraction may result in some initial contraction in real output from $y$ to $y_t$ and a reduction in prices form $P_0$ to $P_1$. ‘Real balance’ effects will overcome the ‘rigidities’, however, and the economy will recover to the full employment level of output and at price level $P_s$, replicating, *with sluggishness only*, the new classical case. See right panel of Figure 4.
The Keynesian case, for comparison purposes can be read from Figure 5.

Figure 5

The AD and AS curves, with increases in $\delta_{CB}$, can be read as dealing with a monetary contraction. The level of output and price levels are lower the higher the service charges, spreads or carrying costs of the Bank. But given the AD function is perfectly vertical because there are, in an anchorless world, no ‘real balance’ effects, there are no price level forces re-equilibrating the economy.\(^{21}\)

\(^{21}\) Bank notes as theoretical anchors, though trivial empirically, are replaced by turning the note issue over to the private banks. If the Bank used its monopoly note issue as a device to enlist the public’s support in the monitoring of private banks (e.g., the substitution to Bank notes from bank deposits), that can be replaced by the willingness of the Bank to accept deposits from any member of the public. The Bank would, of course, pay no interest on such deposits and they would therefore only be used by the public in the case of fear about the liquidity of the banking system as a whole. No nominal anchors are needed for the lender of last resort function by the Bank.
Since, in this simple model, one cannot determine the effects of changes of the level of banking output on the growth rate, no analysis connecting the level of output and 'the' rate of interest is possible. If one argues that the spread is positively related to (say) the overnight rate, the cash rate or the Bank rate, then one could interpret $\delta_{cb}$ in Figure 5 as being equivalent to the overnight rate, where, in the IS diagram, $\delta_{cb}$, the Bank's charge for the provision of liquidity services is replaced by the rate of interest.

One thing is clear. There is no LM function to contemplate because there is no nominal outside anchor. In Keynesian economics monetary policy is the anchor and is always real. It is in this sense that the LM portion of ISLM has vanished.

Colin Rogers
Department of Economics
The University of Adelaide

T. K. Rymes
Department of Economics
Carleton University
Ottawa

February 1998
Appendix

From \( y = \varphi \left( i, y, \frac{\bar{H}}{P} \right) \varphi_1 > 0, \varphi_2 < 1, \varphi_3 > 0 \)

\[
\frac{\bar{H}}{P} = \mu \left( i, y, \frac{\bar{H}}{P} \right) \mu < 0, \mu > 0, \varphi_3 < 1
\]

\[
dy = \frac{[-\varphi_3 \mu_1 - \varphi_2 (1-\mu_3)]}{(1-\varphi_2) \mu_1 + \varphi_1 \mu_2} \frac{\bar{H}}{P^2} dP
\]

such that the denominator being negative and the numerator positive, a lower \( P \) is associated with a higher \( y \). Rewriting as

\[
dy = z \frac{\bar{H}}{P^2} dP
\]

and taking \( Z \) as not a function of \( P \), then

\[
d^2y = Z - \left[ \frac{2\bar{H}}{P^3} \right] (dP)^2
\]

and \( z < 0 \) entails the convexity of \( AD \) in \((P,y)\).

Since

\[
di = \frac{[-(1-\varphi_2) (1-\mu_3) + \varphi_2 \mu_2]}{(1-\varphi_2) \mu_1 + \varphi_1 \mu_2} \frac{\bar{H}}{P^2} dP
\]

with the term in brackets being positive or negative, the relationship between \( i \) and \( P \) cannot be so simply ascertained.